Cap. 8 - Ex. 3

$$\langle V \rangle = \int_0^\infty \int_0^{2\pi} \int_0^{\pi} \psi^* \hat{V} \psi \, dV = \int_0^\infty \int_0^{2\pi} \int_0^{\pi} \psi^* \left(-\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r}\right) \psi) \, r^2 dr \, \text{sen}\theta \, d\theta \, d\varphi$$

$$\psi(r,\theta,\varphi) = R(r) \cdot Y_l(\theta,\varphi)$$

Como os harmônicos esféricos são normalizados, isto é:

$$\int_0^{2\pi} \int_0^{\pi} Y^*_{lm}(\theta,\varphi) Y_{lm}(\theta,\varphi) \, \text{sen}\theta \, d\theta \, d\varphi = 1 \quad \text{(N.A. pg. 161)}$$

$$\langle V \rangle = \frac{-1}{4\pi\varepsilon_0} e^2 \int_0^\infty \frac{1}{r} \left(\frac{1}{\sqrt{\pi}} \frac{1}{a_0^{3/2}} e^{-r/a_0}\right)^2 r^2 dr$$

$$\langle V \rangle = \frac{e^2}{4\pi^2} \frac{1}{a_0^3} \int_0^\infty r \, e^{-2r/a_0} \, dr$$

... façam a integral. Vejam no apêndice. (uuppss... a integral não aparece lá. Vão ter de procurar).