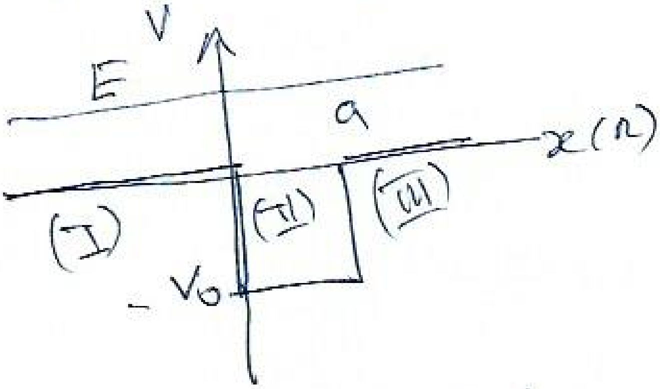
6) Cap. 7 Ex. 6 (1)



$a = 4 \times 10^{-10}$ m

$E = 0{,}7$ eV

$V_0 = ?$

Este problema é equivalente à barreira de potencial (7.3.1), como mostra o ex. 4. Portanto esta é tb a solução do ex 4.

(I) $-\frac{\hbar^2}{2m}\frac{d^2\phi_1}{dx^2} = E\phi_1$; $k_1 = \sqrt{\frac{2mE}{\hbar^2}}$

$\frac{d^2\phi_1}{dx^2} = -k_1^2\phi_1$ $\quad \phi_1(x) = Ae^{+ik_1x} + Be^{-ik_1x}$

(II) $-\frac{\hbar^2}{2m}\frac{d^2\phi_2}{dx^2} - V_0\phi_2 = E\phi_2$; $k_2 = \sqrt{\frac{2m(V_0+E)}{\hbar^2}}$

$\frac{d^2\phi_2}{dx^2} = -k_2\phi_2 \Rightarrow \phi_2(x) = Ce^{ik_2x} + De^{-ik_2x}$

(III) (idêntico a (I)):

$\phi_3(x) = Ee^{ik_1x} + Fe^{-ik_1x}$ (F = 0 pois ñ há partícula vinda de $+\infty$)

– Continuidade de ϕ em $x=0$:

$\phi_1(0) = \phi_2(0) \Rightarrow \boxed{A + B = C + D}$ (1)

– continuidade de $\frac{d\phi}{dx}$ em $x = 0$:

$ik_1(A-B) = ik_2(C-D) \Rightarrow \boxed{A - B = \frac{k_2}{k_1}(C-D)}$ (2)

– continuidade de ϕ em $x = a$

$\boxed{Ce^{ik_2a} + De^{-ik_2a} = Ee^{ik_1a}}$ (3)

- continuidade de $\frac{d\phi}{dx}$ em $x=a$

$$ik_1(Ce^{ik_2a} - De^{-ik_2a}) = ik_1 Ee^{ik_1a}$$

$$\boxed{(Ce^{ik_2a} - De^{-ik_2a}) = \frac{k_1}{k_2} Ee^{ik_1a}} \quad (4)$$

$$(1)+(2) \quad \boxed{2A = C\left(1+\frac{k_2}{k_1}\right) + D\left(1-\frac{k_2}{k_1}\right)} \quad (1')$$

$$(1)-(2) = \boxed{2B = C\left(1-\frac{k_2}{k_1}\right) + D\left(1+\frac{k_2}{k_1}\right)} \quad (2')$$

- Para determinar C e D :

de (3) →

$$C + De^{-2ik_2a} = Ee^{i(k_1-k_2)a} \quad (3')$$

$$\text{de}(4): \quad C - De^{-2ik_2a} = \frac{k_1}{k_2}E\, e^{i(k_1-k_2)a} \quad (4')$$

$$3'+4': \quad \boxed{2C = Ee^{i(k_1-k_2)a}\left(1+\frac{k_1}{k_2}\right)}$$

$$3'-4': \quad \boxed{2De^{-2ik_2a} = Ee^{i(k_1-k_2)a}\left(1-\frac{k_1}{k_2}\right)}$$

~~Substituindo esses resultados em (1') e (2'):~~

$$\cancel{A = \frac{E}{4k_1k_2}\left((k_1+k_2)^2 e^{i(k_1-k_2)a} + \frac{E}{4k_1k_2} e^{i(}\right.}$$

$$A = \frac{C}{2k_1}(k_1+k_2) + \frac{D}{2k_1}(k_1-k_2) \quad (a)$$

$$B = \frac{C}{2k_1}(k_1-k_2) + \frac{D}{2k_1}(k_1+k_2) \quad (b)$$

$$C = \frac{E}{2k_2} e^{i(k_1-k_2)a}(k_1+k_2) \quad (c)$$

$$D = \frac{E}{2k_2}(k_2 - k_1)e^{+i(k_1+k_2)a}$$

de onde:

$$A = \frac{E}{4k_1k_1}\left((k_1+k_2)^2 e^{i(k_1-k_2)a} - (k_1-k_2)^2 e^{i(k_1+k_2)a}\right)$$

$$A = \frac{E e^{ik_1a}}{4k_1k_1}\left((k_1+k_2)^2 e^{-ik_2a} - (k_1-k_2)^2 e^{ik_2a}\right)$$

$$T = \frac{E^*E}{A^*A}$$

$$\frac{A^*A}{E^*E} = \frac{1}{16k_1^2k_2^2}\left[(k_1+k_2)^4 + (k_1-k_2)^4 - (k_1+k_2)^2(k_1-k_2)^2\left(e^{2ik_2a} + e^{-2ik_2a}\right)\right]$$

$$\operatorname{sen}\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \qquad \operatorname{sen}^2\theta = -\frac{1}{4}\left(e^{2i\theta} + e^{-2i\theta} - 2\right)$$

$$\Rightarrow e^{2i\theta} + e^{-2i\theta} = -4\operatorname{sen}^2\theta + 2$$

$$\frac{A^*A}{E^*E} = \frac{1}{16k_1^2k_2^2}\Big((k_1+k_2)^4 + (k_1-k_2)^4 + 4(k_1+k_2)^2(k_1-k_2)^2\operatorname{sen}^2 k_2a$$
$$- 2(k_1+k_2)^2(k_1-k_1)^2\Big)$$

$$k_1 + k_2 = k_s \qquad k_1 - k_2 = k_d$$

$$\frac{A^*A}{E^*E} = \frac{1}{16k_1^2k_2^2}\left(k_s^4 - 2k_s^2k_d^2 + k_d^4 + 4(k_1^2 - k_2^2)^2\operatorname{sen}^2 k_2a\right)$$

$$= \frac{1}{16k_1^2k_2^2}\left((k_s^2 - k_d^2)^2 + 4(k_1^2 - k_2^2)^2\operatorname{sen}^2 k_2a\right)$$

$$K_s^2 = (k_1+k_2)^2 = k_1^2+k_2^2+2k_1k_2$$

$$K_d^2 = (k_1-k_2)^2 = k_1^2+k_2^2-2k_1k_2$$

$$K_s^2 - K_d^2 = 4k_1k_2$$

$$(K_s^2-K_d^2)^2 = 16k_1^2k_2^2$$

$$\frac{A^*A}{E^*E} = \frac{1}{16k_1^2k_2^2}\left(16k_1^2k_2^2 + 4(k_1^2-k_2^2)^2 \operatorname{sen}^2 k_1 a\right)$$

$$= \left[1 + \frac{4(k_1^2-k_2^2)^2}{16k_1^2k_2^2}\operatorname{sen}^2 k_2 a\right]$$

$$T = \frac{E^*E}{A^*A} = \left[1+\frac{(k_1^2-k_2^2)^2}{4k_1^2k_2^2}\operatorname{sen}^2 k_2 a\right]^{-1}$$

p/ $T=1 \Rightarrow \operatorname{sen} k_2 a = 0 \Rightarrow k_2 a = \pi$ (1º valor possível)

$$k_2 = \frac{\pi}{a} = \sqrt{\frac{2m(V_0+E)}{\hbar^2}} \qquad \frac{2m(V_0+E)}{\hbar^2} = \frac{\pi^2}{a^2}$$

$$V_0+E = \frac{\hbar^2\pi^2}{2ma^2} \Rightarrow \boxed{V_0 = \frac{\hbar^2\pi^2}{2ma^2} - E}$$

$$\frac{\hbar^2\pi^2}{2ma^2} = \frac{(\hbar c)^2\pi^2}{2mc^2a^2} = \frac{(2\times10^{-7})^2\cdot 10}{2\times 5\times10^5 \times 16\times10^{-20}} = \frac{4\times10^{-13}}{10^6\times16\times10^{-20}}$$

$$= \frac{1}{4}\times 10 = 2{,}5\ eV \qquad V_0 = 2{,}5 - 0{,}7 = \boxed{1{,}8\ eV}$$