B(M1) e Momentos Magnéticos no Projected Shell Model

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1 Elementos de Matriz de operadores de 1 corpo

(Bohr-Mottelson 3B-2)

$$T = \sum_{1,2>0} <2|T|1> a_2^+ a_1 + \sum_{1<0,2>0} (<2|T|\bar{1}> a_2^+ b_1^+ + <\bar{1}|T|2> b_1 a_2) -$$

$$- \sum_{1,2<0} <\bar{1}|T|\bar{2}> b_2^+ b_1 + \sum_{1<0} <1|T|1>$$

Usando a notação do Hara, onde implicitamente as somas com índice com barra são em valores negativos e trocando os operadores de criação (aniquilação) de buraco por aniquilação (criação) de partícula no orbital time-reversal:

$$b_\nu^+ = a_{\overline{\nu}} \; , \; b_\nu = a_{\overline{\nu}}^+$$

$$T = \sum_{1,2} <2|T|1> a_2^+ a_1 + <2|T|\bar{1}> a_2^+ a_{\bar{1}} + <\bar{1}|T|2> a_{\bar{1}}^+ a_2$$

$$- <\bar{1}|T|\bar{2}> a_{\bar{2}} a_{\bar{1}}^+ + <T>$$

mas:

$$\sum_{1,2} <\bar{1}|T|2> a_{\bar{1}}^+ a_2 = \sum_{1,2} <\bar{2}|T|1> a_{\bar{2}}^+ a_1 \; (trocando\ os\ \acute{i}ndices)$$

$$- \sum_{1,2} <\bar{1}|T|\bar{2}> a_{\bar{2}} a_{\bar{1}}^+ = \sum_{1,2} <\bar{1}|T|\bar{2}> a_{\bar{1}}^+ a_{\bar{2}} = \sum_{1,2} <\bar{2}|T|\bar{1}> a_{\bar{2}}^+ a_{\bar{1}}$$

e

$$T = \sum_{1,2} [<2|T|1> a_2^+ a_1 + <2|T|\bar{1}> a_2^+ a_{\bar{1}} + <\bar{2}|T|1> a_{\bar{2}}^+ a_1 +$$

$$<\bar{2}|T|\bar{1}> a_{\bar{2}}^+ a_{\bar{1}}] + <T>$$

ou, na notação do Hara (N.P. A348):

$$T = <T> + :T:$$

Para o operador magnético M_μ, usando a convenção:
$M_\mu^+ = M_{\bar{\mu}}$, $(T M_\mu T^+) = -M_\mu^+$; T = time reversal operator.
Esta escolha, para um operador ímpar implica por exemplo que para o operador de quadrupolo elétrico (par): $Q_\mu^+ = -Q_{\bar{\mu}}$. (Esta é a convenção usada nos programas do Hara):
$T M_\mu T^+ = -M_{\overline{\mu}}$; $T M_{\bar{\mu}} T^+ = -M_\mu$
$T^+ M_{\bar{\mu}} T = -M_\mu$; $T^+ M_\mu T = -M_{\bar{\mu}}$

$$M_\mu = \sum_{3,4} < 3|M_\mu|4 > a_3^+ a_4 + < 3|M_\mu|\bar{4} > a_3^+ a_{\bar{4}} +$$

$$+ < \bar{3}|M_\mu|4 > a_{\bar{3}}^+ a_4 + < \bar{3}|M_\mu|\bar{4} > a_{\bar{3}}^+ a_{\bar{4}} + < M_\mu >$$

usando as relações (T=time reversal op.):

$$\begin{aligned} T|\nu &> = |\bar{\nu} >;\ T|\bar{\nu} >= -|\nu >;\ < \nu|T^+ =< \bar{\nu}| \\ < &\bar{\nu}|T^+ = - < \nu|;\ TT^+ = T^+T = 1;\ T^2 = -1 \end{aligned}$$

então,

$$< \bar{3}|M_\mu|4 >= - < 3|T^+ M_\mu T|\bar{4} >=< 3|M_{\bar{\mu}}|\bar{4} >$$

$$< \bar{3}|M_\mu|\bar{4} >=< 3|T^+ M_\mu T|4 >= - < 3|M_{\bar{\mu}}|4 >$$

portanto:

$$M_\mu = \sum_{3,4} < 3|M_\mu|4 > a_3^+ a_4 + < 3|M_\mu|\bar{4} > a_3^+ a_{\bar{4}} +$$

$$< 3|M_{\bar{\mu}}|\bar{4} > a_{\bar{3}}^+ a_4 - < 3|M_{\bar{\mu}}|4 > a_{\bar{3}}^+ a_{\bar{4}} + < M_\mu >$$

O próximo passo é aplicar a transformação de quasi-partículas (BCS). A transformação usada nos programas é:

$a_i = u_i c_i - v_i c_{\bar{i}}^+$; $a_{\bar{i}}^+ = v_i c_i + u_i c_{\bar{i}}^+$; $a_{\bar{i}} = u_i c_{\bar{i}} + v_i c_i^+$; $a_i^+ = -v_i c_{\bar{i}} + u_i c_i^+$

(Mudança de notação: para usar a mesma notação utilizada no artigo do Hara (N.P. A348, pg. 200 ,1980): $a \to b$; $c \to a$.

$$M_\mu = \sum_{3,4} \overbrace{< 3|M_\mu|4 > b_3^+ b_4}^{(I)} + \overbrace{< 3|M_\mu|\bar{4} > b_3^+ b_{\bar{4}}}^{(II)} +$$

$$+ \overbrace{< 3|M_{\bar{\mu}}|\bar{4} > b_{\bar{3}}^+ b_4}^{(III)} - \overbrace{< 3|M_{\bar{\mu}}|4 > b_{\bar{3}}^+ b_{\bar{4}}}^{(IV)} + < M_\mu >$$

desenvolvendo os termos $I - IV$:

(I):

$$\sum_{3,4} < 3|M_\mu|4 > b_3^+ b_4 = \sum_{3,4} < 3|M_\mu|4 > (-v_3 a_{\bar{3}} + u_3 a_3^+)(u_4 a_4 - v_4 a_{\bar{4}}^+) =$$

$$= \sum_{3,4} < 3|M_\mu|4 > (-v_3 u_4 a_{\bar{3}} a_4 + v_3 v_4 a_{\bar{3}} a_{\bar{4}}^+ + u_3 u_4 a_3^+ a_4 - u_3 v_4 a_3^+ a_{\bar{4}}^+)$$

(II):

$$\sum_{3,4} < 3|M_\mu|\bar{4} > b_3^+ b_{\bar{4}} = \sum_{3,4} < 3|M_\mu|\bar{4} > (-v_3 a_{\bar{3}} + u_3 a_3^+)(u_4 a_{\bar{4}} + v_4 a_4^+) =$$

$$= \sum_{3,4} < 3|M_\mu|\bar{4} > (-v_3 u_4 a_{\bar{3}} a_{\bar{4}} - v_3 v_4 a_{\bar{3}} a_4^+ + u_3 u_4 a_3^+ a_{\bar{4}} + u_3 v_4 a_3^+ a_4^+)$$

(III):

$$\sum_{3,4} < 3|M_{\bar{\mu}}|\bar{4} > b_{\bar{3}}^{+} b_4 = \sum_{3,4} < 3|M_{\bar{\mu}}|\bar{4} > (v_3 a_3 + u_3 a_{\bar{3}}^{+})(u_4 a_4 - v_4 a_{\bar{4}}^{+}) =$$

$$= \sum_{3,4} < 3|M_{\bar{\mu}}|\bar{4} > (v_3 u_4 a_3 a_4 - v_3 v_4 a_3 a_{\bar{4}}^{+} + u_3 u_4 a_{\bar{3}}^{+} a_4 - u_3 v_4 a_{\bar{3}}^{+} a_{\bar{4}}^{+})$$

(IV):

$$\sum_{3,4} < 3|M_{\bar{\mu}}|4 > b_{\bar{3}}^{+} b_{\bar{4}} = \sum_{3,4} < 3|M_{\bar{\mu}}|4 > (v_3 a_3 + u_3 a_{\bar{3}}^{+})(u_4 a_{\bar{4}} + v_4 a_4^{+}) =$$

$$= \sum_{3,4} < 3|M_{\bar{\mu}}|4 > (v_3 u_4 a_3 a_{\bar{4}} + v_3 v_4 a_3 a_4^{+} + u_3 u_4 a_{\bar{3}}^{+} a_{\bar{4}} + u_3 v_4 a_{\bar{3}}^{+} a_4^{+})$$

2 Elementos de matriz a calcular

Para calcular $< \Psi_{M'}^{I'}|\hat{O}_{\lambda\mu}|\Psi_M^I >$ (notas do Hara) é necessário calcular os elementos de matriz $< \Phi_{k'}|\hat{O}_{\lambda\mu}\hat{P}_{K'-\nu K}^{I}|\Phi_k >$, onde $\Phi_k = a_k^{+}|0> + a_k^{+} a_i^{+} a_j^{+}|0> + ...$ (para núcleos ímpares). No presente caso, nos limitamos a estados de 1 qp: $\Phi_k = a_k^{+}|0>$.

Estes elementos de matriz se reduzem (ver notas do Hara) a elementos do tipo $< \Phi_{k'}|\hat{O}_{\lambda\mu}[\beta]|\Phi_k >$ devido ao processo de projeção.$[\beta]$ ao invés de $[\Omega]$ aparece no caso de simetria axial. Os elementos de matriz a calcular são portanto do tipo:

$$< 0|a_1 \hat{O}_{\lambda\mu}[\beta] a_2^{+}|0 > \equiv < a_1 \hat{O}_{\lambda\mu}[\beta] a_2^{+} >$$

Os elementos de matriz $< a_1 \hat{O}_{\lambda\mu}[\beta] a_2^{+} >$podem ser sempre escritos na forma:

$$< a_1 \hat{O}_{\lambda\mu}[\beta] a_2^{+} > = < \hat{O}_{\lambda\mu}[\beta] > < a_1[\beta] a_2^{+} > + (a_1 \hat{O}_{\lambda\mu}[\beta] a_2^{+})$$

onde a notação $(a_1 \hat{O}_{\lambda\mu}[\beta] a_2^{+})$ implica que nenhuma contração entre a_1 e a_2^{+} é feita. Para verificar, como todo operador $\hat{O}$ contem sempre termos do tipo $a_i a_j$, $a_i^{+} a_j$, etc. tomemos o caso $\hat{O} = a_3 a_4$. Lembrando que elementos do tipo $< a_i a_j a_k[\beta] a_m a_n a_p >$podem ser expandidos em termos de contrações $< a_i[\beta] a_j >, < a_i a_j[\beta] >$, etc., temos:

$$< a_1 a_3 a_4[\beta] a_2^{+} > = < a_3 a_4[\beta] > < a_1[\beta] a_2^{+} > +$$

$$+ < a_1 a_3[\beta] > < a_4[\beta] a_2^{+} > - < a_1 a_4[\beta] > < a_3[\beta] a_2^{+} >$$

$$= < a_3 a_4[\beta] > < a_1[\beta] a_2^{+} > + (a_1 a_3 a_4[\beta] a_2^{+}) = < \hat{O}[\beta] > < a_1[\beta] a_2^{+} > + (a_1 \hat{O}[\beta] a_2^{+})$$

Note-se que o termo constante de $\hat{O}$ $(< \hat{O} >)$ aparece somente no termo $<\hat{O}[\beta] > < a_1[\beta] a_2^{+} >$, portanto $(a_1 \hat{O}[\beta] a_2^{+}) = (a_1 : \hat{O}_{\lambda\mu} : [\beta] a_2^{+})$.

2.1 Cálculo dos elementos $(a_1 : M_\mu : [\beta] a_2^{+})$

Definição das contrações usadas:

$$(a_i[\beta] a_j^{+}) = C_{ij}^1 \; ; (a_{\bar{i}}[\beta] a_{\bar{j}}^{+}) = -C_{ij}^2 \; ; \; (a_i a_j[\beta]) = B_{ij}^1 \; ; \; (a_i a_{\bar{j}}[\beta]) = -(a_{\bar{i}} a_j[\beta]) = B_{ij}^2$$

(I):

$$\sum_{3,4} < 3|M_\mu|4 > \;\; [-v_3 u_4 (a_1 a_{\bar{3}} a_4[\beta] a_2^{+}) + v_3 v_4 (a_1 a_{\bar{3}} a_{\bar{4}}^{+}[\beta] a_2^{+}) +$$

$$+ u_3 u_4 (a_1 a_3^{+} a_4[\beta] a_2^{+}) - u_3 v_4 (a_1 a_3^{+} a_{\bar{4}}^{+}[\beta] a_2^{+})] = \sum - < 3|M_\mu|4 > v_3 u_4 (B_{13}^2 C_{42}^1 + B_{14}^1 C_{32}^2) +$$

$$+\sum_4 < 1|M_\mu|4 > u_1u_4C^1_{42} \equiv \sum_3 < 1|M_\mu|3 > C^1_{32}$$

Note:
$\sum_{34}(a_1a_3^+a_4\ [\beta]a_2^+) \neq 0$ somente quando 3=1, $(a_1a_3^+a_{\bar{4}}^+\ [\beta]a_2^+) = 0$ quaisquer 3,4 e $(a_1a_{\bar{3}}a_{\bar{4}}^+\ [\beta]a_2^+) = -(a_1a_{\bar{4}}^+a_{\bar{3}}\ [\beta]a_2^+) = 0$ (pois o i não tem barra).

(II):

$$\sum_{3,4} < 3|M_\mu|\bar{4} > [-v_3u_4(a_1a_{\bar{3}}a_{\bar{4}}[\beta]a_2^+) - v_3v_4(a_1a_{\bar{3}}a_4^+[\beta]a_2^+)+$$

$$+u_3u_4(a_1a_3^+a_{\bar{4}}[\beta]a_2^+) + u_3v_4(a_1a_3^+a_4^+[\beta]a_2^+)] = \sum_{3,4} - < 3|M_\mu|\bar{4} > (-B^2_{13}C^2_{42} + B^2_{14}C^2_{32})+$$

$$+\sum_{3,4} - < 3|M_\mu|\bar{1} > v_3v_1C^2_{32} - < 1|M_\mu|\bar{3} > u_1u_3C^2_{32}$$

(III):

$$\sum_{3,4} < 3|M_{\bar{\mu}}|\bar{4} > [v_3u_4(a_1a_3a_4[\beta]a_2^+) - v_3v_4(a_1a_3a_{\bar{4}}^+[\beta]a_2^+)+$$

$$u_3u_4(a_1a_3^+a_4[\beta]a_2^+) - u_3v_4(a_1a_3^+a_{\bar{4}}^+[\beta]a_2^+)] = \sum_{3,4} < 3|M_{\bar{\mu}}|\bar{4} > v_3u_4(B^1_{13}C^1_{42} - B^1_{14}C^1_{32})$$

(IV):

$$\sum_{3,4} < 3|M_{\bar{\mu}}|4 > [v_3u_4(a_1a_3a_{\bar{4}}[\beta]a_2^+) + v_3v_4(a_1a_3a_4^+[\beta]a_2^+)+$$

$$u_3u_4(a_1a_3^+a_{\bar{4}}[\beta]a_2^+) + u_3v_4(a_1a_3^+a_4^+[\beta]a_2^+)] = \sum_{3,4} < 3|M_{\bar{\mu}}|4 > v_3u_4(-B^1_{13}C^2_{42} - B^2_{14}C^1_{32})-$$

$$-\sum_3 < 3|M_{\bar{\mu}}|1 > v_1v_3C^1_{32}$$

Para comparar com os resultados do artigo (para o operador Q_μ), vamos calcular o termo $(a_1 : M_\mu + M_{\bar{\mu}} : [\beta]a_2^+)$:

a) termos em $\sum_3\ (I + II + III + IV)$:

$$\sum_3 < 1|M_\mu + M_{\bar{\mu}}|3 > u_1u_3C^1_{32} - (< 3|M_\mu + M_{\bar{\mu}}|\bar{1} > v_3v_1C^2_{32} + < 1|M_\mu + M_{\bar{\mu}}|\bar{3} > u_1u_3C^2_{32})+$$

$$< 3|M_\mu + M_{\bar{\mu}}|1 > v_1v_3C^1_{32} = \sum_3 < 1|M_\mu + M_{\bar{\mu}}|3 > (u_1u_3 + v_1v_3)C^1_{32}-$$

$$- < 1|M_\mu + M_{\bar{\mu}}|\bar{3} > (u_1u_3 + v_1v_3)C^2_{32} =$$

$$= \sum_3 (u_1u_3 + v_1v_3)(< 1|M_\mu + M_{\bar{\mu}}|3 > C^1_{32} - < 1|M_\mu + M_{\bar{\mu}}|\bar{3} > C^2_{32})$$

(Usando a relação:
$< 3|M_\mu + M_{\bar{\mu}}|\bar{1} >= - < \bar{3}|T^+(M_\mu + M_{\bar{\mu}})T|1 >=< \bar{3}|(M_\mu + M_{\bar{\mu}})|1 >= < 1|(M_\mu + M_{\bar{\mu}})|\bar{3} >)$
Para o caso do operador Q_μ, aparece $(u_1u_3 - v_1v_3)$, devido à simetria par do operador Q_μ.

b) termos em $\sum_{3,4}(I + II + III + IV)$:

$$\sum_{3,4} - < 3|M_\mu + M_{\bar{\mu}}|4 > (B^2_{13}C^1_{42} + B^1_{14}C^2_{32})v_3u_4 \ (I)+$$

$$+ < 3|M_\mu + M_{\bar{\mu}}|4 > (B^1_{13}C^2_{42} + B^2_{14}C^1_{32})v_3v_4 \ (IV)+$$

$$+ < 3|M_\mu + M_{\bar{\mu}}|\bar{4} > (B^2_{13}C^2_{42} - B^2_{14}C^2_{32})v_3u_4 \ (II)+$$

$$+ < 3|M_\mu + M_{\bar{\mu}}|\bar{4} > (B^1_{13}C^1_{42} - B^1_{14}C^1_{32})v_3u_4 \ (III)$$

$$= \sum_{3,4} - < 3|M_\mu + M_{\bar{\mu}}|4 > (B^2_{13}C^1_{42} - B^1_{13}C^2_{42})v_3u_4 +$$

$$+ < 3|M_\mu + M_{\bar{\mu}}|4 > (B^2_{14}C^1_{32} - B^1_{14}C^2_{32})v_3u_4 \ (troca\ 3-4)$$

$$+ < 3|M_\mu + M_{\bar{\mu}}|\bar{4} > (B^2_{13}C^2_{42} + B^1_{13}C^1_{42})v_3u_4$$

$$- < 3|M_\mu + M_{\bar{\mu}}|\bar{4} > (B^2_{14}C^2_{32} + B^1_{14}C^2_{32})v_3u_4 \ (troca\ 3-4) =$$

$$= \sum_{3,4}(v_3u_4 - v_4u_3)[< 3|M_\mu + M_{\bar{\mu}}|4 > (B^2_{13}C^1_{42} - B^1_{13}C^2_{42}) - < 3|M_\mu + M_{\bar{\mu}}|\bar{4} > (B^2_{13}C^2_{42} + B^1_{13}C^1_{42})]$$

como no artigo.
Agora, os elementos $(a_1 : M_\mu : [\beta]a_2^+)$:
$\sum_3$:

$$\sum_3 < 1|M_\mu|3 > u_1u_3C^1_{32} - (< 3|M_\mu|\bar{1} > v_1v_3C^2_{32} + < 1|M_\mu|\bar{3} > u_1u_3C^2_{32}) + < 3|M_{\bar{\mu}}|1 > v_1v_3C^1_{32}$$

(usando $< 1|M_\mu|3 > = < 3|M_{\bar{\mu}}|1 >$ e $< 3|M_\mu|\bar{1} > = < \bar{1}|M_{\bar{\mu}}|3 > = < 1|M_\mu|\bar{3} >$)

$$= \sum_3 < 1|M_\mu|3 > u_1u_3C^1_{32} - < 1|M_\mu|\bar{3} > (v_1v_3 + u_1u_3)C^2_{32} + < 1|M_\mu|3 > v_1v_3C^1_{32}$$

$$= \sum_3(v_1v_3 + u_1u_3)[< 1|M_\mu|3 > C^1_{32} - < 1|M_\mu|\bar{3} > C^2_{32}]$$

$\sum_{3,4}$:

$$\sum_{3,4} - < 3|M_\mu|4 > v_3u_4(B^2_{13}C^1_{42} + B^1_{14}C^2_{32}) \ (I)$$

$$+ < 3|M_{\bar{\mu}}|4 > v_3u_4(B^1_{13}C^2_{42} + B^2_{14}C^1_{32}) \ (IV)$$

$$+ < 3|M_\mu|\bar{4} > v_3u_4(B^2_{13}C^2_{42} - B^2_{14}C^2_{32}) \ (II)$$

$$+ < 3|M_{\bar{\mu}}|\bar{4} > v_3u_4(B^1_{13}C^1_{42} - B^1_{14}C^1_{32}) \ (III)$$

(fazendo com que todos os produtos fiquem $B_{13}C_{42}$):

$$- < 3|M_\mu|4 > v_3u_4B^1_{14}C^2_{32} = - < 4|M_\mu|3 > v_4u_3B^1_{13}C^2_{42} = - < 3|M_{\bar{\mu}}|4 > v_4u_3B^1_{13}C^2_{42}$$

$$< 3|M_{\bar{\mu}}|4 > v_3u_4B^2_{14}C^1_{32} =< 4|M_{\bar{\mu}}|3 > v_4u_3B^2_{13}C^1_{42} =< 3|M_{\mu}|4 > v_4u_3B^2_{13}C^1_{42}$$

$$< 3|M_{\mu}|\bar{4} > v_3u_4B^2_{14}C^2_{32} = - < 4|M_{\mu}|\bar{3} > v_4u_3B^2_{13}C^2_{42} = - < 3|M_{\mu}|\bar{4} > v_4u_3B^2_{13}C^2_{42}$$

$$< 3|M_{\bar{\mu}}|\bar{4} > v_3u_4B^1_{14}C^1_{32} = - < 4|M_{\bar{\mu}}|\bar{3} > v_4u_3B^1_{13}C^1_{42} = - < 3|M_{\bar{\mu}}|\bar{4} > v_4u_3B^1_{13}C^1_{42}$$

$$\sum_{3,4} -(v_3u_4 - v_4u_3)[(< 3|M_{\mu}|4 > B^2_{13}C^1_{42}- < 3|M_{\bar{\mu}}|4 > B^1_{13}C^2_{42})+$$

$$+(< 3|M_{\mu}|\bar{4} > B^2_{13}C^2_{42}+ < 3|M_{\bar{\mu}}|\bar{4} > B^1_{13}C^1_{42})]$$

Finalmente:

$$(a_1 : M_{\mu} : [\beta]a^+_2) = \sum_3 (u_1u_3 + v_1v_3)[< 1|M_{\mu}|3 > C^1_{32}- < 1|M_{\mu}|\bar{3} > C^2_{32}]-$$

$$-\sum_{3,4}(v_3u_4 - v_4u_3)[(< 3|M_{\mu}|4 > B^2_{13}C^1_{42}- < 3|M_{\bar{\mu}}|4 > B^1_{13}C^2_{42})+$$

$$(< 3|M_{\mu}|\bar{4} > B^2_{13}C^2_{42}+ < 3|M_{\bar{\mu}}|\bar{4} > B^1_{13}C^1_{42})]$$

Colocando os índices (1,2) como surgem nas somatórias:
a) $\sum_3$

$$(\bar{1}M_{\mu}2) = \sum_3(u_1u_3 + v_1v_3)(< 1|M_{\bar{\mu}}|3 > C^2_{32}+ < 1|M_{\bar{\mu}}|\bar{3} > C^1_{32})$$

$$(1M_{\mu}\bar{2}) = \sum_3(u_1u_3 + v_1v_3)(< 1|M_{\mu}|3 > C^2_{32}+ < 1|M_{\bar{\mu}}|\bar{3} > C^1_{32}$$

$$(\bar{1}M_{\mu}\bar{2}) = \sum_3(u_1u_3 + v_1v_3)(- < 1|M_{\bar{\mu}}|3 > C^1_{32}+ < 1|M_{\mu}|\bar{3} > C^2_{32}$$

.Note que as relações $(1M_{\bar{\mu}}\bar{2}) = (\bar{1}M_{\mu}2)$ e $(\bar{1}M_{\mu}\bar{2}) = -(1M_{\bar{\mu}}2)$ são verificadas (Ao menos para os termos em $\sum_3$).

b)$\sum_{3,4}$ (Usando as relações: $B^1_{1\bar{2}} = B^2_{12}$; $B^2_{1\bar{2}} = -B^1_{12}$):

$$(\bar{1}M_{\mu}2) = -\sum_{3,4}(u_3v_4 - v_4u_3)[(< 3|M_{\mu}|4 > B^1_{13}C^1_{42}+ < 3|M_{\bar{\mu}}|4 > B^2_{13}C^2_{42})+$$

$$+(< 3|M_{\mu}|\bar{4} > B^1_{13}C^2_{42}- < 3|M_{\bar{\mu}}|\bar{4} > B^2_{13}C^1_{42})]$$

$$(1M_{\mu}\bar{2}) = -\sum_{3,4}(u_3v_4 - v_4u_3)[(< 3|M_{\mu}|4 > B^2_{13}C^2_{42}+ < 3|M_{\bar{\mu}}|4 > B^1_{13}C^1_{42})+$$

$$+(< 3|M_{\bar{\mu}}|\bar{4} > B^1_{13}C^2_{42}- < 3|M_{\mu}|\bar{4} > B^2_{13}C^1_{42})]$$

($(1M_{\bar{\mu}}\bar{2}) = (\bar{1}M_{\mu}2)$!!)

$$(\bar{1}M_{\mu}\bar{2}) = \sum_{3,4}(u_3v_4 - v_4u_3)[(< 3|M_{\mu}|4 > B^1_{13}C^2_{42}- < 3|M_{\bar{\mu}}|4 > B^2_{13}C^1_{42})+$$

$$-(< 3|M_\mu|\bar{4} > B^1_{13}C^1_{42} + < 3|M_{\bar{\mu}}|\bar{4} > B^2_{13}C^2_{42})]$$

($(\bar{1}M_\mu\bar{2}) = -(1M_{\bar{\mu}}2)$!!)

Agora, calculando explicitamente os temos para $\mu = 0$ e $\mu \neq 0$:

a) termos em $\sum_3$:

$\mu = 0$:

usando a definição:

$(1M_02) =< 1|M_0|3 > C^1_{32} \equiv < 1|M_0|3 > C_1$,temos:

$(1M_02) =< 1|M_0|3 > C_1$

$(1M_0\bar{2}) =< 1|M_0|3 > C_2$

$(\bar{1}M_02) =< 1|M_0|3 > C_2$

$(\bar{1}M_0\bar{2}) = - < 1|M_0|3 > C_1$

ou, na forma matricial, como são construidos estes elementos no programa $T_1 =< 1|M_0|3 >$:

$$\begin{bmatrix} T_1 & 0 \\ 0 & -T_1 \end{bmatrix} \cdot \begin{bmatrix} C_1 & C_2 \\ -C_2 & C_1 \end{bmatrix} = \begin{bmatrix} T_1C_1 & T_1C_2 \\ T_1C_2 & -T_1C_1 \end{bmatrix} \equiv \begin{bmatrix} (1M_02) & (1M_0\bar{2}) \\ (\bar{1}M_02) & (\bar{1}M_0\bar{2}) \end{bmatrix}$$

$\mu \neq 0$:

$(1|M_\mu|2) = - < 1|M_\mu|\bar{3} > C_2$

$(1|M_\mu|\bar{2}) = + < 1|M_\mu|\bar{3} > C_1$

$(\bar{1}|M_\mu|2) = + < 1|M_\mu|\bar{3} > C_1$

$(\bar{1}|M_\mu|\bar{2}) = + < 1|M_\mu|\bar{3} > C_2$

$$\begin{bmatrix} 0 & T_1 \\ T_{\bar{1}} & 0 \end{bmatrix} \cdot \begin{bmatrix} C_1 & C_2 \\ -C_2 & C_1 \end{bmatrix} = \begin{bmatrix} -T_1C_2 & T_1C_1 \\ T_{\bar{1}}C_1 & T_{\bar{1}}C_2 \end{bmatrix} \equiv \begin{bmatrix} (1|M_\mu|2) & (1|M_\mu|\bar{2}) \\ (\bar{1}|M_\mu|2) & (\bar{1}|M_\mu|\bar{2}) \end{bmatrix}$$

(idênticos aos termos equivalentes para Q_μ no programa.)

Antes de ver os termos de M_μ com $\sum_{3,4}$, vejamos como se comportam os equivalentes para Q_μ ($\mu = \pm 1$):

$$(1|Q_\mu|2) = -(v_3u_4 + v_4u_3)(< 3|Q_\mu|4 > B^2_{13}C^1_{42} + < 3|Q_{\bar{\mu}}|4 > B^1_{13}C^2_{42})$$

$$(1|Q_\mu|\bar{2}) = -(v_3u_4 + v_4u_3)(< 3|Q_\mu|4 > B^2_{13}C^2_{42} - < 3|Q_{\bar{\mu}}|4 > B^1_{13}C^1_{42})$$

$$(\bar{1}|Q_\mu|2) = -(v_3u_4 + v_4u_3)(< 3|Q_\mu|4 > B^1_{13}C^1_{42} - < 3|Q_{\bar{\mu}}|4 > B^2_{13}C^2_{42})$$

$$(\bar{1}|Q_\mu|\bar{2}) = -(v_3u_4 + v_4u_3)(< 3|Q_\mu|4 > B^1_{13}C^2_{42} + < 3|Q_{\bar{\mu}}|4 > B^2_{13}C^1_{42})$$

$[(\bar{1}|Q_\mu|\bar{2}) = (1|Q_{\bar{\mu}}|2)$; $(\bar{1}|Q_\mu|2) = -(1|Q_{\bar{\mu}}|\bar{2})]$

$$\begin{bmatrix} B_1 & B_2 \\ -B_2 & B_1 \end{bmatrix} \begin{bmatrix} 0 & T_1 \\ -T_1 & 0 \end{bmatrix} = \begin{bmatrix} -B_2T_1 & B_1T_1 \\ -B_1T_1 & -B_2T_1 \end{bmatrix} \cdot \begin{bmatrix} C_1 & C_2 \\ -C_2 & C_1 \end{bmatrix} =$$

$$\begin{bmatrix} -T_1(B_2C_1 + B_1C_2\ ) & -T_1(B_2C_2 - B_1C_1) \\ T_1(B_2C_2 - B_1C_1) & -T_1(B_2C_1 + B_1C_2) \end{bmatrix} = - \begin{bmatrix} (1|Q_0|2) & (1|Q_0|\bar{2}) \\ (\bar{1}|Q_0|2) & (\bar{1}|Q_0|\bar{2}) \end{bmatrix}$$

O termo correspondente à primeira multiplicação (BT) é o W_1 no AT1A.

Termos em $\mu = 0$ ($\sum_{3,4}$) para o operador $M\mu$:

$$\begin{bmatrix} B_1 & B_2 \\ -B_2 & B_1 \end{bmatrix} \cdot \begin{bmatrix} 0 & T_1 \\ T_1 & 0 \end{bmatrix} = \begin{bmatrix} B_2T_1 & B_1T_1 \\ B_1\,T_1 & -B_2T_1 \end{bmatrix} \cdot \begin{bmatrix} C_1 & C_2 \\ -C_2 & C_1 \end{bmatrix} =$$

$$\begin{bmatrix} T_1(B_2C_1 - B_1C_2) & T_1(B_2C_2 - B_1C_1) \\ T_1(B_2C_2 + B_1C_1) & -T_1(B_2C_1 - B_1C_2) \end{bmatrix} = - \begin{bmatrix} (1|M_0|2) & (1|M_0|\bar{2}) \\ (\bar{1}|M_0|2) & (\bar{1}|M_0|\bar{2}) \end{bmatrix}$$

$(1|M_0|2) = - < 3|M_0|4 > (B_2C_1 - B_1C_2)$

$(1|M_0|\bar{2}) = - < 3|M_0|4 > (B_2C_2 + B_1C_1)$

$(\bar{1}|M_0|2) = - < 3|M_0|4 > (B_2C_2 + B_1C_1)$

$(\bar{1}|M_0|\bar{2}) = + < 3|M_0|4 > (B_2C_1 - B_1C_2)$

Termos em $\mu \neq 0$ ($\sum_{3,4}$)

$(1|M_\mu|2) = -(< 3|M_\mu|\bar{4} > B_2C_2 + < 3|M_{\bar{\mu}}|\bar{4} > B_1C_1)$

$(1|M_\mu|\bar{2}) = -(< 3|M_{\bar{\mu}}|\bar{4} > B_1C_2 - < 3|M_\mu|\bar{4} > B_2C_1)$
$(\bar{1}|M_\mu|2) = -(< 3|M_\mu|\bar{4} > B_1C_2 - < 3|M_{\bar{\mu}}|\bar{4} > B_2C_1)$
$(\bar{1}|M_\mu|\bar{2}) = +(< 3|M_\mu|\bar{4} > B_1C_1 + < 3|M_{\bar{\mu}}|\bar{4} > B_2C_2)$
$(\bar{1}|M_\mu|\bar{2}) = -(1|M_{\bar{\mu}}|2)$; $(\bar{1}|M_\mu|2) = (1|M_{\bar{\mu}}|\bar{2})$

$$\begin{bmatrix} B_1 & B_2 \\ -B_2 & B_1 \end{bmatrix} \cdot \begin{bmatrix} T_{\bar{1}} & 0 \\ 0 & -T_1 \end{bmatrix} = \begin{bmatrix} B_1T_{\bar{1}} & -B_2T_1 \\ -B_2T_{\bar{1}} & -B_1T_1 \end{bmatrix} \cdot \begin{bmatrix} C_1 & C_2 \\ -C_2 & C_1 \end{bmatrix} =$$

$$\begin{bmatrix} T_{\bar{1}}B_1C_1 + T_1B_2C_2 & T_{\bar{1}}B_1C_2 - T_1B_2C_1 \\ T_1B_1C_2 - T_{\bar{1}}B_2C_1 & -(T_1B_1C_1 + T_{\bar{1}}B_2C_2) \end{bmatrix} = - \begin{bmatrix} (1|M_\mu|2) & (1|M_\mu|\bar{2}) \\ (\bar{1}|M_\mu|2) & (\bar{1}|M_\mu|\bar{2}) \end{bmatrix}$$

Para colocar no programa, devemos ainda substituir $\bar{\mu}$ por $-\mu$. ($M_{\bar{\mu}} = (-)^\mu M_{-\mu}$):
no caso de $\mu = 0$, nada muda:

$$T_{11}(\mu = 0) \equiv \begin{bmatrix} T_1 & 0 \\ 0 & T_1 \end{bmatrix}; \; T_{02} \equiv \begin{bmatrix} 0 & T_1 \\ T_1 & 0 \end{bmatrix}$$

$\mu \neq 0$:

$$T_{11} \equiv \begin{bmatrix} 0 & T_\mu \\ -T_{-\mu} & 0 \end{bmatrix}; \; T_{02} \equiv \begin{bmatrix} -T_{-\mu} & 0 \\ 0 & -T_\mu \end{bmatrix}$$

Agora, os termos $<M_\mu[\beta] > C^1_{12}$. (São as contrações que faltavam nos termos do tipo $v_3u_4(a_1a_3a_4[\beta]a_2^+)$:

$$< M_\mu[\beta] >= \sum_{3,4} v_3u_4[< 3|M_\mu|4 > B^2_{34} - < 3|M_\mu|\bar{4} > B^1_{34} -$$

$$- < 3|M_{\bar{\mu}}|\bar{4} > B^1_{34} - < 3|M_{\bar{\mu}}|4 > B^2_{34}] =$$

$$= \sum_{3>0;\, 4\geq 3} vu_4 \left[(< 3|M_\mu|4 > - < 3|M_{\bar{\mu}}|4 > B^2_{34} - (< 3|M_\mu|\bar{4} > - < 3|M_{\bar{\mu}}|\bar{4} > B^1_{34}\right] +$$

$$+ \sum_{4>0;\, 3\geq 4} v_3u_4 \left[< 3|M_\mu - M_{\bar{\mu}}|4 > B^2_{34} - < 3|M_\mu - M_{\bar{\mu}}|\bar{4} > B^1_{34}\right] \text{ (troca indices)}$$

$$= \sum_{3>0;\, 4\geq 3} \{v_3u_4 \left[< 3|M_\mu - M_{\bar{\mu}}|4 > B^2_{34} - < 3|M_\mu - M_{\bar{\mu}}|\bar{4} > B^1_{34}\right] +$$

$$+v_4u_3 \left[< 4|M_\mu - M_{\bar{\mu}}|3 > B^2_{43} - < \bar{4}|M_\mu - M_{\bar{\mu}}|3 > B^1_{43}\right]\}/(1 + \delta_{34})$$

$$= \sum_{3>0;\, 4\geq 3} \{v_3u_4 \left[< 3|M_\mu - M_{\bar{\mu}}|4 >)B^2_{34} - < 3|M_\mu - M_{\bar{\mu}}|\bar{4} >)B^1_{34}\right] -$$

$$-v_4u_3 \left[< 3|M_\mu - M_{\bar{\mu}}|4 >)B^2_{34} - < 3|M_\mu - M_{\bar{\mu}}|\bar{4} >)B^1_{34}\right]\}/(1 + \delta_{34})$$

$$= \sum_{3>0;\, 4\geq 3} (v_3u_4 - v_4u_3) \left[< 3|M_\mu - M_{\bar{\mu}}|4 >)B^2_{34} - < 3|M_\mu - M_{\bar{\mu}}|\bar{4} >)B^1_{34}\right] /(1 + \delta_{34})$$

No programa, este termo é calculado em SPUR. É tomada a soma dos termos diagonais da matriz $T_{02}B$.

a) $\mu = 0$

$$\begin{bmatrix} 0 & T_1 \\ T_1 & 0 \end{bmatrix} \cdot \begin{bmatrix} B_1 & B_2 \\ -B_2 & B_1 \end{bmatrix} = \begin{bmatrix} -T_1B_2 & T_1B_1 \\ T_1B_1 & T_1B_2 \end{bmatrix}$$

os termos diagonais são do tipo $- < 3|M_0|4 > B_2 + < 3|M_0|4 > B_2 = 0$

b) $\mu \neq 0$

$$\begin{bmatrix} T_{\bar{\mu}} & 0 \\ 0 & -T_{\mu} \end{bmatrix} \cdot \begin{bmatrix} B_1 & B_2 \\ -B_2 & B_1 \end{bmatrix} = \begin{bmatrix} T_{\bar{\mu}}B_1 & T_{\bar{\mu}}B_2 \\ T_{\mu}B_2 & -T_{\mu}B_1 \end{bmatrix}$$

termos diagonais: $< 3|M_{\bar{\mu}}|\bar{4} > B_1 - < 3|M_{\mu}|\bar{4} > B_1 =< 3|M_{\mu} - M_{\bar{\mu}}|\bar{4} > B_1$!

Notas sobre MQASI.

1) $SEL(N, IMU, MIX) \equiv < I|M_{\mu}|J >$ $(N = I, J)$ só é calculado para $J \leq I$ (DEFSET). A outra parte deve ser obtida por simetria:

a) $\mu = 0 :< I|M_0|J >=< J|M_0^+|I >=< J|M_0|I >$

b) $\mu \neq 0 :\ < I|M_{\mu}|\bar{J} >= - < \bar{I}|T^+ M_{\mu} T|J >=< \bar{I}|M_{\bar{\mu}}|J >=< J|M_{\bar{\mu}}^+|\bar{I} >=< J|M_{\mu}|\bar{I} >$

Os elementos T_{11} e T_{02} devem ser construidos (matrizes) de forma a gerar os termos correspondentes às contrações $(1 : M_{\mu} : [\beta]2^+)$, $(1 : M_{\mu} : [\beta]\bar{2}^+)$, etc. (ver pag. 11)

a) $\mu = 0$

$T_{11}(I, J) = (u_i u_j + v_i v_j) < 3|M_0|4 >$

$T_{11}(I + NL, J + NL) = -(u_i u_j + v_i v_j) < 3|M_0|4 >= -T_{11}(I, J)$

os elementos que faltam são portanto:

$T_{11}(J, I) = T_{11}(I, J)$

$T_{11}(J + NL, I + NL) = T_{11}(I + NL, J + NL)$

$T_{02}(I, J + NL) = -(u_i v_j - v_i u_j) < I|M_0|J >$

$T_{02}(I + NL, J) = -(u_i v_j - v_i u_j) < I|M_0|J >= T_{02}(I, J + NL)$

e os faltantes:

$T_{02}(J + NL, I) = -T_{02}(I, J + NL)$

$T_{02}(J, I + NL) = -T_{02}(I + NL, J)$ (o sinal de - vem pela troca I - J nos termos $u_i v_j$.

b) $\mu \neq 0$

$T_{11}(I, J + NL) = (u_i u_j + v_i v_j) < 3|M_{\mu}|\bar{4} >$

$T_{11}(I + NL, J) = -(u_i u_j + v_i v_j) < 3|M_{-\mu}|\bar{4} >$

e os faltantes:

$T_{11}(J + NL, I) = T_{11}(I, J + NL)$

$T_{11}(J, I + NL) = T_{11}(I + NL, J)$

$T_{02}(I, J) = (u_i v_j - v_i u_j) < 3|M_{-\mu}|\bar{4} >$

$T_{02}(I + NL, J + NL) = (u_i v_j - v_i u_j) < 3|M_{\mu}|\bar{4} >$

e os faltantes:

$T_{02}(J, I) = -T_{02}(I, J)$

$T_{02}(J + NL, I + NL) = -T_{02}(I + NL, J + NL)$

2.2 Operador para momento angular na base esférica.

Para poder reduzir as projeções a um *lado* do operador nos elementos de matriz $(< \Psi^{I'}_{M'}|P^{I'}\hat{O}P^I|\Psi^I >=< \Psi^{I'}_{M'}|\hat{O}P^I|\Psi^I >)$, os operadores para momento angular devem ser os operadores tensoriais $Y_{\lambda\mu}$, $\lambda = 1$ (na verdade um operador vetorial pois $\lambda = 1$). Estes operadores se relacionam com J_+, J_- (raizing and lowering op.) por:

$J_1 = -J_+$, $J_{-1} = J_-$, $J_0 = J_z$

e os elementos de matriz para estes operadores (ver Bohr Mottelson vol. 1 ap. A?):

$$\Psi^I_M = \sum_k F^I_k P^I_{MK}|\Phi_k > e\ \Psi^{I'}_{M'} = \sum_{k'} F^{I'}_{k'} P^{I'}_{M'K'}|\Phi_{k'} >$$

$$< \Psi^{I'}_{M'}|T_{\lambda\mu}|\Psi^I_M >= \sum_{k,k'} F^I_k F^{I'}_{k'} < \Phi_{k'}|P^{I'+}_{M'K'} T_{\lambda\mu} P^I_{MK}|\Phi_k >= \sum_{k,k'} F^I_k F^{I'}_{k'} < \Phi_{k'}|P^{I'}_{K'M'} T_{\lambda\mu} P^I_{MK}|\Phi_k >$$

$$P^{I'}_{M'K'} T_{\lambda\mu} = \sum_{\nu, J, M", K"} < JM"\lambda\mu|I'M' >< JK"\lambda\mu|I'K' > T_{\lambda\mu} P^J_{M"K"}$$

$P^J_{K"M"} P^I_{MK} = \delta_{ij} P^I_{K"K}$

$< \Psi^{I'}_{M'}|T_{\lambda\mu}|\Psi^I_M >=$

$$\sum_{k,k'} \sum_{\nu, J, M", K"} < JM"\lambda\mu|I'M' >< JK"\lambda\mu|I'K' > F^I_k F^{I'}_{k'} < \Phi_{k'}|T_{\lambda\mu} P^J_{K"M"} P^I_{MK}|\Phi_k >$$

$$< \Psi^{I'}_{M'}|T_{\lambda\mu}|\Psi^{I}_{M} >=< IM\lambda\mu|I'M' > \sum_{\nu k k'} < IK'-\nu\lambda\mu|I'K' > F^{I}_{k}F^{I'}_{k'} < \Phi_{k'}|T_{\lambda\mu}P^{I}_{K'-\nu K}|\Phi_{k} >$$

$< I'M'|T_{\lambda\mu}|IM >= (2I'-1)^{-1/2} < IM\lambda\mu|I'M' >< I'||T_{\lambda}||I >\equiv$
$< IM\lambda\mu|I'M' > \sum$
$< I'||T_{\lambda}||I >= \sum \cdot\sqrt{(2I'+1)}$
$B(M1, I' \to I) = | < I'||T_{\lambda}||I > |^2 \cdot \frac{1}{2I'+1} = \sum^2$
$B(M1, I \to I') = B(M1, I' \to I)\frac{2I'+1}{2I+1} = \sum^2 \cdot \frac{2I'+1}{2I+1}$

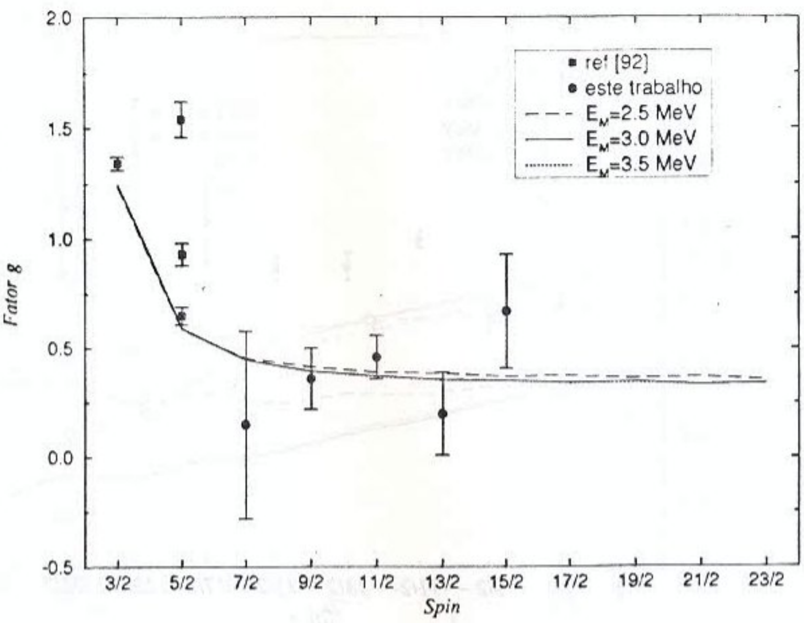

Figure 1: Resultados - Tese do N.H. Medina