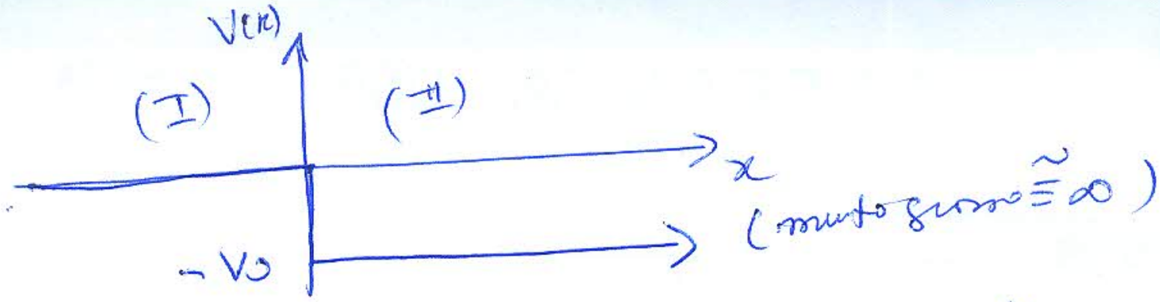
1)



(análogo ao 7.2.2, bastando trocar $V_0 \to -V_0$)

(I) $-\frac{\hbar^2}{2m}\frac{d^2\phi_1}{dx^2} = E\phi_1 \ (V=0) \Rightarrow \frac{d^2\phi_1}{dx^2} = -\frac{2mE}{\hbar^2}\phi_1 = -k_1^2\phi_1$

$$\Rightarrow \boxed{\phi_1(x) = A e^{ik_1x} + B e^{-ik_1x}}$$ 1,0

(II) $-\frac{\hbar^2}{2m}\frac{d^2\phi_2}{dx^2} - V_0\phi_2 = E\phi_2 \Rightarrow \frac{d^2\phi_2}{dx^2} = -\frac{2m}{\hbar^2}(V_0+E)\phi_2 = -k_2^2\phi_2$

$$\Rightarrow \boxed{\phi_2(x) = C e^{ik_2x} + D e^{-ik_2x}}$$ 1,0

$D=0$ (ñ há elétrons vindo de $+\infty$).

$\phi_1(0) = \phi_2(0) \quad = A + B = C$

$\phi_1'(0) = \phi_2'(0) \Rightarrow A - B = \frac{k_2}{k_1}C$

(+) : $\boxed{2A = \left(1+\frac{k_2}{k_1}\right)C}$ 0,5   (−) $\boxed{2B = \left(1-\frac{k_2}{k_1}\right)C}$ 0,5

$A e^{i(k_1x - \frac{E}{\hbar}t)} \longrightarrow$ onda (fluxo) incidente

$B e^{-i(k_1x + \frac{E}{\hbar}t)} \longleftarrow$ onda (fluxo) refletida

$C e^{i(k_2x - \frac{E}{\hbar}t)} \longrightarrow$ onda (fluxo) transmitida

coef. de reflexão : $R = \frac{v_r|\psi_r|^2}{v_i|\psi_i|^2} = \frac{B^*B}{A^*A} = \boxed{\frac{(k_1-k_2)^2}{(k_1+k_2)^2}}$ 1,0

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② a)

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + \frac{1}{2}kx^2\phi = E\phi$$

$$-\frac{d^2\phi}{dx^2} + \frac{mk}{\hbar^2}x^2\phi = \frac{2mE}{\hbar^2}\phi \quad ; \quad \frac{mk}{\hbar^2} = \frac{m^2k}{\hbar^2 m} = \frac{m^2\omega^2}{\hbar^2} = \alpha^2$$

$$\phi = Axe^{-\alpha x^2/2} \qquad \frac{d\phi}{dx} = A\left(e^{-\alpha x^2/2} - \alpha x^2 e^{-\alpha x^2/2}\right)$$

$$\frac{d^2\phi}{dx^2} = A\left(-\alpha x - 3\alpha x + \alpha^2x^3\right)e^{-\alpha x^2/2} =$$

$$= -\underbrace{Axe^{-\alpha x^2/2}}_{\phi(x)}(3\alpha - \alpha^2x^2)$$

$$\phi(x)(3\alpha - \alpha^2x^2) + \alpha^2x^2\phi = \frac{2mE}{\hbar^2}\phi$$

$$\frac{2mE}{\hbar^2} = 3\alpha \qquad E = \frac{3}{2}\alpha\frac{\hbar^2}{m} = \frac{3}{2}\frac{m\omega}{\hbar}\cdot\frac{\hbar^2}{m} = \frac{3}{2}\hbar\omega$$

2,0

$$\boxed{E = \frac{3}{2}\hbar\omega} \qquad E_{osc} = \left(n+\frac{1}{2}\right)\hbar\omega \Rightarrow \boxed{n=1}$$

$$A^*A\int_{-\infty}^{\infty} xe^{-\alpha x^2/2}\cdot xe^{-\alpha x^2/2}dx = A^*A\int_{-\infty}^{\infty} x^2e^{-\alpha x^2}dx = 2A^*A\int_0^{\infty} x^2e^{-\alpha x^2}dx =$$

$$= 2A^*A\,\frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}} = 1 \qquad A^*A = 2\sqrt{\frac{\alpha^3}{\pi}}$$

1,0

$$\boxed{A = \sqrt{2}\cdot\frac{\alpha^{3/4}}{\pi^{1/4}}}$$

(3)

$$\psi(x,t) = A \cos(kx)\, e^{-i\omega t}$$

$$\cos kx = \frac{e^{ikx} + e^{-ikx}}{2}$$

1,0

1,0

$$\psi(x,t) = \frac{A}{2} e^{i(kx-\omega t)} + \frac{A}{2} e^{-i(kx+\omega t)}$$

$\psi_i(x,t)$ → incidente

← $\psi_r(x,t)$ refletida.

Coef reflexão $R = \frac{v_r|\psi_r|^2}{v_i|\psi_i|^2} = \frac{A^2}{A^2} = 1$

1,0